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Heterotic string field theory

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Abstract: We construct the Neveu-Schwarz sector of heterotic string field theory using the large Hilbert space of the superghosts and the multi-string products of bosonic closed string field theory. No picture-changing operators are required as in Wess-Zumino-Witten-like open superstring field theory. The action exhibits a novel kind of nonpolynomiality: in addition to terms necessary to cover missing regions of moduli spaces, new terms arise from the boundary of the missing regions and its subspaces. We determine the action up to quintic order and a subset of terms to all orders.

Keywords: Superstrings and Heterotic Strings, String Field Theory.

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Contents

1. Introduction

String field theory is one possible approach to the construction of a nonperturbative formulation of string theory. There are two known Lorentz-covariant bosonic string field theories. One is an open string field theory [1] with a Chern-Simons-like cubic action, and the other one is a closed string field theory [2, 3] with nonpolynomial interactions which are necessary to cover the moduli spaces of punctured spheres. There is also an open-closed bosonic string field theory [4], and there is a conjectural vacuum string field theory [5].

The construction of superstring field theory is complicated because of superghost pictures. For the Neveu-Schwarz (NS) sector of open superstrings, the difficulties have been resolved by working in the large Hilbert space that includes the zero mode of the fermionic superghost ξ. The resulting Wess-Zumino-Witten-like (WZW-like) action [6] contains an infinite number of regular contact terms. The great advantage of this formulation is that no picture-changing operators appear in the action. It is also possible to write heterotic and, perhaps, type-II string field theories if we allow the insertion of picture-changing operators in the action [7]. The resulting structure, however, is complicated and noncanonical because one must give a prescription to deform the closed string vertices and prevent the collision of picture-changing operators.[1]

In this paper we construct a Lorentz-covariant string field theory for the NS sector of heterotic strings [9]. We work in the large Hilbert space and use the nonpolynomial structure of bosonic closed string field theory to produce a cover of the moduli spaces of punctured spheres. Gauge invariance further requires elementary interactions associated with the *boundary* of the missing regions of the moduli spaces and with lower-dimensional subspaces of this boundary. As in the case of WZW-like open superstring field theory, no

[1] Open superstring field theory actions with explicit picture-changing operators were considered in [8].

insertions of picture-changing operators are necessary. Our work was partially motivated by the desire to have calculable closed superstring field theories to investigate conjectures concerning twisted tachyons in orbifold backgrounds [10].

2. Open superstring and closed bosonic string field theories

In this section we review the structure of WZW open superstring field theory and bosonic closed string field theory, focusing on the aspects necessary for our construction of heterotic string field theory. We begin with the open superstring field theory [6].

A general off-shell open string field configuration in the GSO-even NS sector corresponds to a Grassmann *even* state $|\Phi\rangle$ of ghost number zero and picture number zero in the combined conformal field theory (CFT) of matter, ghosts, and superghosts. This string field lives in the 'large Hilbert space' that contains the zero mode ξ_0 of the field ξ. For the superghosts ξ, η, and ϕ, the assignments of ghost number (G) and picture number (P) are as follows:

$$\xi : \; G = -1\,, \; P = 1\,, \qquad \eta : \; G = 1\,, \; P = -1\,, \qquad e^{q\phi} : \; G = 0\,, \; P = q\,. \tag{2.1}$$

The string field theory action takes the following form:

$$S = \frac{1}{2g^2}\Big\langle\Big\langle (e^{-\Phi}Qe^{\Phi})\,(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt\,(e^{-t\Phi}\partial_t e^{t\Phi})\,\{\,(e^{-t\Phi}Qe^{t\Phi}),\,(e^{-t\Phi}\eta_0 e^{t\Phi})\,\}\Big\rangle\Big\rangle\,, \tag{2.2}$$

where $\{A,\,B\} \equiv AB + BA$, Q is the BRST operator, and $\eta_0 = \oint dz\,\eta(z)$. This action is defined using Witten's star product [1] by expanding all exponentials in formal Taylor series preserving the order of all operators. To cubic order one finds

$$S = \frac{1}{2g^2}\Big\langle\Big\langle \frac{1}{2}\,(Q\Phi)\,(\eta_0\Phi) \; + \frac{1}{6}\,(Q\Phi)\Big(\,\Phi\,(\eta_0\Phi) - (\eta_0\Phi)\,\Phi\,\Big)\Big\rangle\Big\rangle + \mathcal{O}(\Phi^4)\,. \tag{2.3}$$

The full action is invariant under gauge transformations with gauge parameters Λ and Ω:

$$\delta e^{\Phi} = (Q\Lambda)\,e^{\Phi} + e^{\Phi}\,(\eta_0\Omega)\,. \tag{2.4}$$

The gauge invariance can be proven using, among others, $\{Q,\eta_0\} = 0$ and $Q^2 = \eta_0^2 = 0$. The equation of motion for the string field is $\eta_0\,(e^{-\Phi}Qe^{\Phi}) = 0$, which, to linearized order, reduces to

$$Q\eta_0|\Phi\rangle = 0\,. \tag{2.5}$$

Using the Ω gauge invariance one can choose the gauge $\xi_0|\Phi\rangle = 0$, and thus write $|\Phi\rangle = \xi_0|\widehat{\Phi}\rangle$, where $\eta_0|\widehat{\Phi}\rangle = 0$. Here $|\widehat{\Phi}\rangle$ is a state in the 'small Hilbert space', the space that does not include the zero mode of ξ. In the gauge $|\Phi\rangle = \xi_0|\widehat{\Phi}\rangle$, the linearized equation of motion reduces to $Q|\widehat{\Phi}\rangle = 0$. This equation of motion is satisfied when the vertex operator corresponding to the state $|\widehat{\Phi}\rangle$ takes the form of $ce^{-\phi}V_M$, where V_M is a matter primary with dimension 1/2. The corresponding state $|\Phi\rangle = \xi_0|\widehat{\Phi}\rangle$ has ghost number and picture number zero, as expected. In the GSO even sector V_M is Grassmann odd. Since ξ_0, c, and $e^{-\phi}$ are all Grassmann odd, $|\Phi\rangle$ is Grassmann even.

Let us now turn to the second ingredient of the construction. The bosonic closed string field theory action [2] is given by

$$S = -\frac{2}{\alpha'}\left(\frac{1}{2}\langle\Psi,\, Q\Psi\,\rangle + \frac{1}{3!}\,\kappa\langle\Psi,[\Psi,\,\Psi]\rangle + \frac{1}{4!}\,\kappa^2\langle\Psi,[\Psi,\,\Psi,\,\Psi]\rangle + \dots\right). \tag{2.6}$$

The closed string field $|\Psi\rangle$ has ghost number two and satisfies the subsidiary conditions $(b_0 - \bar{b}_0)|\Psi\rangle = (L_0 - \bar{L}_0)|\Psi\rangle = 0$. There is an infinite set of string products, all of which are graded-*commutative*. For example, the lowest product satisfies $[A,\,B] = (-1)^{AB}[B,\,A]$, where string states in the exponent represent their Grassmann property, 0 (mod 2) for Grassmann even states and 1 (mod 2) for Grassmann odd states. The linear inner product is defined by $\langle A\,, B\rangle = \langle A|c_0^-|B\rangle$, where $\langle A|$ is the BPZ conjugate of $|A\rangle$, and $c_0^- = \frac{1}{2}(c_0 - \bar{c}_0)$. Some important identities satisfied by the lowest product, the inner product, and the BRST operator Q are

$$\begin{aligned} \langle A,\,B\rangle &= (-1)^{(A+1)(B+1)}\langle B,\,A\rangle\,, \\ \langle QA,\,B\rangle &= (-1)^{A}\langle A,\,QB\rangle\,, \\ \langle[A,\,B]\,,\,C\rangle &= (-1)^{A+B}\langle A,[B,\,C]\rangle\,. \end{aligned} \tag{2.7}$$

The products, together with the inner product, define fully graded-commutative multilinear forms:

$$\{B_1,\,B_2,\,\dots,\,B_n\} \equiv \langle B_1,[B_2,\,\dots,\,B_n]\rangle\,. \tag{2.8}$$

The fundamental relation satisfied by the closed string products takes the form:

$$\begin{aligned} 0 = {} & Q[B_1,\,\dots,\,B_n] + \sum_{i=1}^{n}(-1)^{(B_1+\dots+B_{i-1})}[B_1,\,\dots,\,QB_i,\,\dots,\,B_n] + \\ & + \sum_{\{i_l,\,j_k\}} \sigma(i_l,\,j_k)[B_{i_1},\,\dots,\,B_{i_l},[B_{j_1},\,\dots,\,B_{j_k}]]\,. \end{aligned} \tag{2.9}$$

The sum in the second line runs over all different splittings of the set $\{1,2,\dots,n\}$ into a first group $\{i_1,\dots,i_l\}$ and a second group $\{j_1,\dots,j_k\}$, where $l \geq 1$ and $k \geq 2$. Two splittings are the same if the corresponding first groups contain the same set of integers regardless of their order. The sign factor $\sigma(i_l,\,j_k)$ is the sign picked up when one rearranges the sequence $\{Q,\,B_1,\,B_2,\,\dots\,,\,B_n\}$ into the sequence $\{B_{i_1},\,\dots,\,B_{i_l},\,Q,\,B_{j_1},\,\dots,\,B_{j_k}\}$ taking into account the Grassmann property of the various objects. For $n = 2$, the identity is

$$0 = Q[B_1,\,B_2] + [QB_1,\,B_2] + (-1)^{B_1}[B_1,\,QB_2]\,. \tag{2.10}$$

This is the derivation property of Q. One might have expected a minus sign in front of the first term on the right-hand side, but the sign is absent because the string product carries an insertion of $b_0 - \bar{b}_0$. For $n = 3$, the identity (2.9) is

$$\begin{aligned} 0 = {} & Q[B_1,\,B_2,\,B_3] + [QB_1,\,B_2,\,B_3] + (-1)^{B_1}[B_1,\,QB_2,\,B_3] + \\ & + (-1)^{B_1+B_2}[B_1,\,B_2,\,QB_3] + (-1)^{B_1}[B_1,[B_2,\,B_3]] + \\ & + (-1)^{B_2(1+B_1)}[B_2,[B_1,\,B_3]] + (-1)^{B_3(1+B_1+B_2)}[B_3,[B_1,\,B_2]]\,. \end{aligned} \tag{2.11}$$

Because of insertions of b ghosts, Q is no longer a derivation. The violation of the derivation property of Q is related to the violation of the Jacobi identity.

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The ghost number of a string product is not the sum of the ghost number of each state. For a product with n input states we have

$$G([B_1,\, B_2,\, \ldots,\, B_n]) = -2(n-2) - 1 + \sum_{i=1}^{n} G_i\,, \tag{2.12}$$

where G_i is the ghost number of B_i.

3. The construction of heterotic string field theory

The string field theory that we will construct can be formulated for any consistent background of heterotic string theory, namely, for any conformal field theory with the following structure of holomorphic and antiholomorphic sectors. The holomorphic sector is comprised of an $N = 1$ superconformal matter theory with central charge $c = 15$, reparameterization ghosts (b, c) with central charge $c = -26$, and superghosts (ξ, η, ϕ) with $c = 11$. The antiholomorphic sector is not supersymmetric and consists of a matter theory with $\bar{c} = 26$ and reparametrization ghosts $(\bar{b}, \bar{c})$ with $\bar{c} = -26$. The full theory has an NS sector and a Ramond (R) sector, depending on the boundary conditions of fermions on the supersymmetric side. In this paper we focus on the NS sector and assume that this sector has been truncated to the GSO even states. Presumably, a formulation that describes both GSO even and odd states can be obtained with small modifications, as discussed in detail in [11].

The open superstring field was defined to be Grassmann even and had both ghost number and picture number zero. In bosonic open string field theory the string field is Grassmann odd, has ghost number one, and carries no picture. By tensoring the two string fields, the resulting closed string field V should be Grassmann odd and have ghost number one and picture number zero:

$$V:\ \text{Grassmann odd}\,, \quad G = 1\,, \quad P = 0\,. \tag{3.1}$$

We work in the large state space of the superconformal sector. Physical vertex operators corresponding to $|V\rangle$ in the gauge $\xi_0|V\rangle = 0$ take the form of $\mathcal{V} = \xi\, c\bar{c}\, V_M\, e^{-\phi}$. Here V_M is a Grassmann odd matter primary operator with dimensions (1/2, 1). Since $\mathcal{V}$ is Grassmann odd, so is the string field. The normalization of correlators in the full CFT for a flat spacetime background is given by

$$\begin{aligned}\langle\langle \xi(w_1)\; e^{-2\phi(w_2)}\, c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\, e^{ip\cdot X(z,\bar{z})}\rangle\rangle &= \\ = 2(2\pi)^D \delta^{(D)}(p)|z_1 - z_2|^2|z_1 - z_3|^2|z_2 - z_3|^2\,,\end{aligned} \tag{3.2}$$

where $D = 10$ is the dimension of spacetime. Nonvanishing correlators require total ghost number five and total picture number minus one. Since we will write the action using the inner product $\langle\,\cdot\,,\,\cdot\,\rangle$ which contains an insertion of c_0^-, each term in the action must have ghost number four and picture number minus one.

In the full CFT we have a BRST operator Q that is nilpotent and, as usual, satisfies the relations $\{Q, b(z)\} = T(z)$ and $\{Q, \bar{b}(\bar{z})\} = \bar{T}(\bar{z})$, with $T, \bar{T}$ the total stress tensor of the conformal field theory. Since we are dealing with a closed string theory, the following subsidiary conditions apply:

$$(b_0 - \bar{b}_0)|V\rangle = (L_0 - \bar{L}_0)|V\rangle = 0\,. \tag{3.3}$$

The inner product is $\langle A\,, B\rangle = \langle A|c_0^-|B\rangle$, formally the same one we had for bosonic closed strings. The string products are obtained by integrating forms over certain subsets of the ordinary moduli spaces of punctured Riemann spheres. The forms have no superghost insertions, while insertions of b antighosts are present in exactly the same way as in bosonic closed string field theory. This implies that the string products in heterotic strings satisfy the identity (2.9). The identities in (2.7) also hold. For simplicity, we denote the zero mode of the field η by η itself. Then we have

$$\langle \eta A,\, B\rangle = (-1)^A \langle A,\, \eta B\rangle\,. \tag{3.4}$$

Since the string products contain no superghost insertions, η is a derivation:

$$0 = \eta[B_1,\, \ldots,\, B_n] + \sum_{i=1}^{n} (-1)^{(B_1+\ldots+B_{i-1})}[B_1,\, \ldots,\, \eta B_i,\, \ldots,\, B_n]\,. \tag{3.5}$$

3.1 Quadratic and cubic terms in the action

The kinetic term for the heterotic string field theory action is expected to give the linearized field equation $Q\eta|V\rangle = 0$, which in the gauge $\xi_0|\,V\,\rangle = 0$ reduces to the conventional BRST cohomology problem in the small Hilbert space. We expand the action in powers of the gravitational constant κ:

$$S = \frac{2}{\alpha'} \sum_{n=2}^{\infty} \kappa^{n-2} S_n\,, \tag{3.6}$$

and we write S_2 as

$$S_2 = \frac{1}{2}\langle \eta V,\, QV\rangle\,. \tag{3.7}$$

As required, the total ghost number and total picture number for the operators in the above inner product are four and minus one, respectively. Using (2.7) and (3.4) the variation of S_2 is given by

$$\delta S_2 = \langle \delta V,\, Q\eta V\rangle = \langle Q\eta V,\, \delta V\rangle\,, \tag{3.8}$$

which yields the correct linearized equation of motion. The kinetic term is invariant under the transformations

$$\delta_\Lambda^{(0)} V = Q\Lambda\,, \qquad \delta_\Omega^{(0)} V = \eta\Omega\,. \tag{3.9}$$

The cubic interaction requires the lowest string product. Since V is Grassmann odd and the string products are graded-commutative, the product $[V,\, V]$ vanishes. On the other hand, $[V,\, QV]$ does not vanish, and it has ghost number two, as can be seen from (2.12). We write S_3 as

$$S_3 = \frac{1}{3!}\langle \eta V,\, [V,\, QV]\rangle\,, \tag{3.10}$$

or, using the multilinear form, as

$$S_3 = \frac{1}{3!}\{\eta V,\, V,\, QV\}\,. \tag{3.11}$$

Since the sum of pictures of the operators involved must be minus one, there is just one factor of η. This will be the case for all terms in the action. It is instructive to see how the above cubic term reduces to the expected correlator for physical states. First we note that $\{A,\, B,\, C\} = \langle\langle A(\infty)\, B(0)\, C(1)\rangle\rangle$ for physical states, where $\langle\langle \ldots \rangle\rangle$ denotes correlator. Writing $|V\rangle = \xi_0|\widehat{V}\rangle$, where $Q|\widehat{V}\rangle = 0$ and $|\widehat{V}\rangle$ has picture minus one, we see that

$$S_3 = \frac{1}{3!}\langle\langle\{\eta_0, V(\infty)\}V(0)\{Q, V(1)\}\rangle\rangle = \frac{1}{3!}\langle\langle\widehat{V}(\infty)\xi\widehat{V}(0)\oint\frac{dz}{2\pi i z}\,X(z)\,\widehat{V}(1)\rangle\rangle\,, \tag{3.12}$$

where $X(z) = \{Q,\, \xi(z)\}$ is the picture-changing operator and the contour of the integral encircles $z = 1$ counterclockwise. The expression on the right-hand side, using (3.2), reduces to the small Hilbert space correlator of $\widehat{V}(\infty)$, $\widehat{V}(0)$, and $\lim_{z\to 1} X(z)\widehat{V}(1)$, the last of which is the vertex operator in the zero picture. This is the expected result.

To see how the gauge transformations are modified by the addition of the cubic interaction we first consider the general variation of S_3. Using the following formulas

$$\begin{aligned}
0 &= \{QB_1,\, B_2,\, B_3\} + (-1)^{B_1}\{B_1,\, QB_2,\, B_3\} + (-1)^{B_1+B_2}\{B_1,\, B_2,\, QB_3\}\,,\\
0 &= \{\eta\, B_1,\, B_2,\, B_3\} + (-1)^{B_1}\{B_1,\, \eta\, B_2,\, B_3\} + (-1)^{B_1+B_2}\{B_1,\, B_2,\, \eta\, B_3\}\,,
\end{aligned} \tag{3.13}$$

which follow from (2.7), (2.8), (2.10), (3.4), and (3.5), a short calculation gives

$$\delta S_3 = \frac{1}{2}\{\delta V,\, \eta V,\, QV\} = \frac{1}{2}\langle\delta V, [\eta V,\, QV]\rangle\,. \tag{3.14}$$

It follows that the gauge variation $\delta_\Lambda^{(0)} S_3$ is given by

$$\delta_\Lambda^{(0)} S_3 = \frac{1}{2}\langle Q\Lambda,\, [\eta V,\, QV]\rangle\,. \tag{3.15}$$

Using $[\eta V,\, QV] = -Q[\eta V,\, V] - [Q\eta V,\, V]$, we obtain

$$\delta_\Lambda^{(0)} S_3 = -\frac{1}{2}\langle Q\Lambda, [Q\eta V,\, V]\rangle = -\frac{1}{2}\langle Q\eta V, [V,\, Q\Lambda]\rangle\,. \tag{3.16}$$

Making use of (3.8) we deduce that the above variation of the cubic term can be cancelled against a variation of the quadratic term by modifying the gauge transformation:

$$\delta_\Lambda V = Q\Lambda + \frac{\kappa}{2}[V,\, Q\Lambda] + \mathcal{O}(\kappa^2)\,. \tag{3.17}$$

The gauge transformation generated by η works out in a similar way:

$$\delta_\Omega^{(0)} S_3 = \frac{1}{2}\langle\eta\Omega, [\eta V,\, QV]\rangle\,. \tag{3.18}$$

Using $[\eta V,\, QV] = -\eta[V,\, QV] - [V,\, Q\eta V]$, we find

$$\delta_\Omega^{(0)} S_3 = -\frac{1}{2}\langle\eta\Omega, [V,\, Q\eta V]\rangle = -\frac{1}{2}\langle Q\eta V, [\eta\Omega,\, V]\rangle\,. \tag{3.19}$$

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This variation is also cancelled by modifying the gauge transformation:

$$\delta_\Omega V = \eta\Omega + \frac{\kappa}{2}[\eta\Omega,\, V] + \mathcal{O}(\kappa^2)\,. \tag{3.20}$$

To cubic order the action is

$$S = \frac{2}{\alpha'}\left[\frac{1}{2}\langle \eta V,\, QV\rangle + \frac{\kappa}{3!}\langle \eta V, [V,\, QV]\rangle\right] + \mathcal{O}(\kappa^2)\,, \tag{3.21}$$

and the equation of motion takes the form:

$$Q\eta V + \frac{\kappa}{2}\,[\,\eta V,\, QV\,] + \mathcal{O}(\kappa^2) = 0\,. \tag{3.22}$$

We have computed the on-shell scattering amplitude of three gravitons using the action (3.21). We have confirmed that κ is the gravitational constant by comparing our result with the properly normalized amplitude in equation (12.4.14) of [12].

3.2 Higher-order terms

In this subsection we first make a few remarks on the general structure of higher-order terms in the string field theory action. We then list our results for the quartic and quintic terms, together with the corresponding gauge transformations. Finally, we determine certain classes of terms in the action to all orders.

We have already learned that each term in the action has one factor of η. Using (2.7), (2.8), the graded-commutativity of the multilinear forms, and possibly (2.9), we can always write any interaction as an inner product of ηV with a string field build with string products of V's and QV's:

$$\langle\, \eta V, [\ldots[\ldots[\ldots]\ldots[\ldots]\ldots]]\rangle\,. \tag{3.23}$$

Consider how many times QV can appear in the terms of $\mathcal{O}(V^{N+1})$. The minimum number of products is one, in which case the product has N entries and the term is

$$\langle \eta V, [V,\, (QV)^{N-1}]\rangle \equiv \langle \eta V, [V,\, \underbrace{QV,\, QV,\, \ldots,\, QV}_{N-1}]\rangle\,. \tag{3.24}$$

Here we introduced the shorthand notation $(QV)^{N-1}$ for a collection of $N-1$ entries of QV. There cannot be more than one V since any product with multiple V's vanishes by graded-commutativity. The $(N-1)$ QV's are in fact necessary for the ghost number to work. One can easily see from (2.12) that the string product has ghost number two when there are $(N-1)$ QV's. The maximum possible number of products in the terms of $\mathcal{O}(V^{N+1})$ is $N-1$. In this case, the interaction term takes the following form:

$$\langle \eta V,\ \underbrace{[V,[V,[V,\,\ldots\,[V,}_{N-1}\, QV]]]\ldots]\rangle\,. \tag{3.25}$$

To investigate the general constraint from ghost number, consider a product with N inputs:

$$[B_1,\, B_2,\, \ldots,\, B_N]\,. \tag{3.26}$$

We can introduce additional products by writing additional pairs of brackets inside the outer brackets. If we add one pair of brackets, we have two products, one with N_1 inputs and one with N_2 inputs, where $N_1 + N_2 = N + 1$. In fact, each time we add a pair of brackets the number of inputs increases by one unit. For a term built with k products with input numbers $N_1,\, N_2,\, N_3,\, \ldots,\, N_k$, the total number of inputs is $N + k - 1$. Using (2.12) the ghost number of the term in question is

$$G = \sum_{j=1}^{k}(-2N_j+3)+\sum_{i=1}^{N} G_i = -2(N+k-1)+3k+\sum_{i=1}^{N} G_i = -2N+k+2+\sum_{i=1}^{N} G_i\,. \quad (3.27)$$

Since we need $G = 2$, we learn that $\sum_{i=1}^{N} G_i = 2N - k$. With V's and QV's the only possible inputs, we must use k copies of V and $N - k$ copies of QV. In summary, S_{N+1} *consists of terms of the form* $\langle\, \eta V,\, R_k\,\rangle$, *where* $1 \leq k \leq N-1$, *and* R_k *is a state built using* k *string products,* k *copies of* V, *and* $N - k$ *copies of* QV. *Each product has exactly one entry that is* V.

Let us look at the quartic and quintic terms explicitly. The above analysis tells us that there are two possible terms in the quartic part S_4 of the action: the first uses a single product with three inputs, and the second uses two products each of which has two inputs. We have found by computation that gauge invariance uniquely determines the coefficients for these terms to be

$$S_4 = \frac{1}{4!}\Big(\langle \eta V, [V,\, QV,\, QV]\rangle + \langle \eta V, [V, [V,\, QV]]\rangle\Big)\,. \quad (3.28)$$

Geometrically, the first term is a correlator integrated over the subspace $\mathcal{V}_{0,4}$ of the moduli space $\mathcal{M}_{0,4}$ of four-punctured spheres. The subspace $\mathcal{V}_{0,4}$ is determined canonically in closed string field theory using minimal area metrics, and it has an explicit description in terms of polyhedra associated with Jenkins-Strebel quadratic differentials. In the computation of string amplitudes, this term is necessary to produce a cover of the moduli space $\mathcal{M}_{0,4}$. The second term in (3.28) is a correlator integrated over a part of the boundary of $\mathcal{V}_{0,4}$. Although it is required by gauge invariance, it does not contribute to on-shell scattering amplitudes when the vertex operators take the form of $\xi\, c\bar{c}\, e^{-\phi}$ times a primary operator in the matter sector.

With the inclusion of S_4, the gauge transformations of the string field acquire terms quadratic in V. One finds

$$\delta_\Lambda V \,=\, Q\Lambda + \frac{\kappa}{2}[V,\, Q\Lambda] + \kappa^2\Big(\frac{1}{6}[V,\, QV,\, Q\Lambda] + \frac{1}{12}[V, [V,\, Q\Lambda]]\Big) + \mathcal{O}(\kappa^3)\,, \quad (3.29)$$

$$\delta_\Omega V \,=\, \eta\Omega + \frac{\kappa}{2}[\eta\Omega,\, V] + \kappa^2\Big(\frac{1}{3}[\eta\Omega,\, QV,\, V] + \frac{1}{12}[[\eta\Omega,\, V]\,, V]\Big) + \mathcal{O}(\kappa^3)\,. \quad (3.30)$$

The gauge transformations are not uniquely determined since we can redefine the gauge parameters. We have fixed this ambiguity by the requirement that Λ always appear as $Q\Lambda$ and Ω always appear as $\eta\Omega$. The correction to the equation of motion follows from the general variation of the quartic terms:

$$\delta\big(\langle \eta V, [V,\, QV,\, QV]\rangle + \langle \eta V, [V, [V,\, QV]]\rangle\big) =$$

$$= 4\langle \delta V, [QV,\, QV,\, \eta V]\rangle - 2\langle \delta V, [V, [QV,\, \eta V]]\rangle + \\ +4\langle \delta V, [\eta V, [V,\, QV]]\rangle + 2\langle \delta V, [V, [V,\, Q\eta V]]\rangle\,. \tag{3.31}$$

We have also determined the quintic part S_5 of the action completely. This time there are four terms: one term with one product, two inequivalent terms with two products, and one term with three products. Their coefficients are again uniquely determined by gauge invariance. We found

$$S_5 = \frac{1}{5!}\Big(\langle \eta V, [V,\, QV,\, QV,\, QV]\rangle + \langle \eta V, [V, [V,\, QV,\, QV]]\rangle + \\ + 3\langle \eta V, [V,\, QV, [V,\, QV]]\rangle + \langle \eta V, [V, [V, [V,\, QV]]]\rangle\Big)\,. \tag{3.32}$$

The gauge transformations $\delta_\Lambda V$ and $\delta_\Omega V$ acquire the following extra terms:

$$\delta_\Lambda^{(3)} V = \frac{1}{4!}\Big([V,\, QV,\, QV,\, Q\Lambda] + [V,\, QV, [\,V,\, Q\Lambda]] + [[V,\, QV]\,,\, V,\, Q\Lambda]\Big)\,, \tag{3.33}$$

$$\delta_\Omega^{(3)} V = \frac{1}{8}[\eta\Omega,\, QV,\, QV,\, V] + \frac{1}{12}[[\eta\Omega,\, QV,\, V]\,, V] + \\ +\frac{1}{8}[\eta\Omega, [QV,\, V]\,,\, V] + \frac{1}{24}[[\eta\Omega, V]\,, QV,\, V]\,, \tag{3.34}$$

where $\delta_\Lambda^{(m)} V$ and $\delta_\Omega^{(m)} V$ are defined by

$$\delta_\Lambda V = \sum_{m=0}^{\infty} \kappa^m\, \delta_\Lambda^{(m)} V\,, \qquad \delta_\Omega V = \sum_{m=0}^{\infty} \kappa^m\, \delta_\Omega^{(m)} V\,. \tag{3.35}$$

While we have not determined all terms in the string field theory action and gauge transformations, we have completely determined the terms in the action that are built with one string product and with two string products. We have also derived the terms in the gauge transformations that are built with one string product. For the action we find

$$S_n = \frac{1}{n!}\left(\langle \eta V, [V,\, (QV)^{n-2}]\rangle + \sum_{m=0}^{n-4} \binom{n-2}{m} \langle \eta V, [V,\, (QV)^m, [V,\, (QV)^{n-m-3}]]\rangle\right) + \\ +\mathcal{O}(Q^{n-4})\,, \tag{3.36}$$

and for the gauge transformations we find

$$\delta_\Lambda^{(m)} V = \frac{1}{(m+1)!}[V,\, (QV)^{m-1},\, Q\Lambda] + \mathcal{O}(Q^{m-1})\,, \tag{3.37}$$

$$\delta_\Omega^{(m)} V = \frac{m}{(m+1)!}[\eta\Omega,\, (QV)^{m-1},\, V] + \mathcal{O}(Q^{m-2})\,. \tag{3.38}$$

We present the derivation of (3.36), (3.37), and (3.38) in the appendix. We found that gauge invariance under either $\delta_\Lambda^{(m)} V$ or $\delta_\Omega^{(m)} V$ alone determines the coefficients in (3.36) uniquely and gives the same set of coefficients. We regard this as evidence that a unique gauge-invariant action exists to all orders.

Let us conclude the section by making some comments on the geometrical meaning of the various interactions in the action. In bosonic closed string field theory the moduli space $\mathcal{M}_{0,n}$ of n-punctured spheres is covered by the contributions of an elementary vertex $\mathcal{V}_{0,n}$ and Feynman diagrams built with lower-order vertices and propagators. One writes

$$\mathcal{M}_{0,n} = \mathcal{V}_{0,n} \cup \mathcal{R}^{(1)}_{0,n} \cup \mathcal{R}^{(2)}_{0,n} \cup \cdots \cup \mathcal{R}^{(n-3)}_{0,n}\,, \tag{3.39}$$

where $\mathcal{R}^{(I)}_{0,n}$ denotes the region of moduli space obtained by Feynman graphs with I propagators. As indicated, the maximum number of propagators is $n-3$. The Feynman graphs in $\mathcal{R}^{(n-3)}_{0,n}$ are built using only the three string vertex, and all the moduli arise from the propagators, each of which carries two moduli, length and twist angle. In bosonic closed string field theory, the n-th order contribution to the action is simply a correlator over $\mathcal{V}_{0,n}$. This is all that is needed for gauge invariance. In heterotic theory elementary vertices arise from *all* components of (3.39): in addition to the correlator integrated over $\mathcal{V}_{0,n}$, there are correlators integrated over subspaces obtained by collapsing *all* propagators of $\mathcal{R}^{(I)}_{0,n}$. Each time we collapse a propagator we lose the length parameter, but the twist-angle parameter survives. For $\mathcal{R}^{(1)}_{0,n}$ we collapse the single propagator and we obtain a subset of $\mathcal{M}_{0,n}$ of codimension one. For $\mathcal{R}^{(2)}_{0,n}$ we collapse the two propagators and we obtain a subset of $\mathcal{M}_{0,n}$ of codimension two. For $\mathcal{R}^{(n-3)}_{0,n}$ we collapse the $n-3$ propagators and we obtain a subset of $\mathcal{M}_{0,n}$ of codimension $n-3$, namely, a subspace of dimension $n-3$, which is half of the dimension of $\mathcal{M}_{0,n}$. As a result, the dimensionalities of the subspaces of $\mathcal{M}_{0,n}$ that define the vertices range from the top dimension to half of that dimension. The highest and lowest dimensional terms correspond, in the case of $\mathcal{M}_{0,N+1}$, to (3.24) and (3.25), respectively. If we attempted to build vertices associated with such lower dimensional subspaces in bosonic string field theory, they would vanish for string fields of ghost number two, which is the ghost number of the classical string field.

For four-punctured spheres we have $\mathcal{M}_{0,4} = \mathcal{V}_{0,4} \cup \mathcal{R}^{(1)}_{0,4}$. The two components correspond to the two terms in (3.28). For quintic terms we have $\mathcal{M}_{0,5} = \mathcal{V}_{0,5} \cup \mathcal{R}^{(1)}_{0,5} \cup \mathcal{R}^{(2)}_{0,5}$. The first term in (3.32) corresponds to an integral over $\mathcal{V}_{0,5}$, the next two terms are integrals over $\mathcal{R}^{(1)}_{0,5}$ with the propagator collapsed, and the last term is an integral over $\mathcal{R}^{(2)}_{0,5}$ with both propagators collapsed. At each order the general set of vertices that we have presented in (3.36) correspond to the top subspace $\mathcal{V}_{0,n}$ and $\mathcal{R}^{(1)}_{0,n}$ with one propagator collapsed. Note, however, that not all Feynman graphs produce elementary interactions because, as we discussed earlier, each string product must have one entry of V for the ghost number to work. If we define an internal vertex to be one all of whose legs are not external legs of the Feynman graph, the rule can be briefly stated as follows: graphs with internal vertices do not generate elementary interactions.

A canonical construction of the subspaces $\mathcal{V}_{0,n}$ is obtained in terms of restricted n-faced polyhedra: polyhedra in which each face has perimeter 2π and all nontrivial closed paths along the edges are larger than or equal to 2π. The punctured sphere associated with a polyhedron is built by attaching a semi-infinite cylinder to each face of the polyhedron. The surfaces obtained from $\mathcal{R}^{(I)}_{0,n}$ by collapsing all I propagators are in fact restricted polyhedra in which I nontrivial, non-intersecting closed paths along the edges have length exactly equal to 2π. This gives a unified description of all the moduli spaces associated with heterotic string vertices in terms of $\mathcal{V}_{0,n}$ and its subspaces.

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4. Open questions and discussion

The most important remaining task is finding full closed-form expressions for the string field theory action and the gauge transformations. Our partial results should give useful clues in this search. The WZW open superstring field theory uses exponentials of string fields. This is natural because the string field is Grassmann even and the open string star product is associative. With the rich structure of homotopy Lie-algebra products that appears in closed string field theory, there may exist useful generalizations of the exponential function. Since the number of QV's in the action increases with the order of the interaction and QV is Grassmann even, some special function built with QV may play a role in the construction. The two gauge transformations $\delta_\Lambda V$ and $\delta_\Omega V$ in heterotic string field theory take rather different forms. Each one separately appears to determine the action completely. It remains to show the existence of a unique action which is invariant under both gauge transformations to all orders.

Our construction is reminiscent of 'heterosis', the process of combining a right-moving superstring with a left-moving bosonic string [9]. To see this, we recall that one may be able to view bosonic closed string field theory as a kind of tensor product of two bosonic open string field theories [13]. In our case we combine an open superstring field theory and an open bosonic string field theory, the result being a closed string field theory with a new algebraic structure. While the algebraic structures of both open and closed bosonic string field theories were naturally understood in terms of master actions that satisfy the Batalin-Vilkovisky master equation, such understanding is still lacking for the WZW open superstring theory and, of course, for the present heterotic string field theory.

It would also be of interest to write the NS-NS sector of a type-II superstring field theory. An intriguing generalization of the WZW open superstring theory was discussed in [14], where the large Hilbert space was used for both holomorphic and antiholomorphic sectors, and a cubic covariantized light-cone theory was proposed. For a Lorentz covariant theory higher-point interactions of the form (2.8) are necessary to cover the moduli spaces of punctured spheres. If we try to construct such a term with N string fields of ghost number and picture number zero and operators η, $\bar{\eta}$, and Q which preserve the conditions (3.3), the constraints from ghost and picture numbers require $2N-4$ insertions of BRST operators. Since this number is greater than the number of string fields for large N, and we cannot have more than one Q acting on a string field due to nilpotency, our construction does not seem to extend to the type-II case in a straightforward way.

Despite its nonpolynomiality, the WZW open superstring field theory is not significantly harder to use than the cubic open bosonic string field theory. This is because the higher point functions carry no moduli, and no insertions of picture-changing operators are necessary. We expect that the heterotic string field theory proposed here will not be significantly harder to use than bosonic closed string field theory. We believe that computations of tachyon potentials and investigations of spacetime background changes are now feasible using heterotic string field theory.

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A. Constraints from gauge invariance

We expand the gauge transformations of S in powers of κ:

$$\delta_\Lambda S = \frac{2}{\alpha'} \sum_{n=0}^{\infty} \kappa^n \, (\delta_\Lambda S)_n \,, \qquad \delta_\Omega S = \frac{2}{\alpha'} \sum_{n=0}^{\infty} \kappa^n \, (\delta_\Omega S)_n \,. \tag{A.1}$$

We can further classify the terms in S_n, $(\delta_\Lambda S)_n$, and $(\delta_\Omega S)_n$ by the number of BRST operators. The maximum number of Q's in S_n is $n-2$ for $n \geq 3$. Terms with $(n-2)$ Q's and terms with $(n-3)$ Q's can be written as

$$S_n = f_n \langle \eta V, [V,\, (QV)^{n-2}] \rangle + \sum_{m=0}^{n-4} f_{n,m} \langle \eta V, [V,\, (QV)^m, [V,\, (QV)^{n-m-3}]] \rangle + \mathcal{O}(Q^{n-4}) \,, \tag{A.2}$$

for $n \geq 3$, where f_n and $f_{n,m}$ are coefficients to be determined by gauge invariance. The maximum number of Q's in $\delta_\Lambda^{(m)} V$ is m for $m \geq 1$, and that in $\delta_\Omega^{(m)} V$ is $m-1$ for $m \geq 1$. Terms with the maximum number of Q are written as

$$\delta_\Lambda^{(m)} V = g_{m+1} [V,\, (QV)^{m-1},\, Q\Lambda] + \mathcal{O}(Q^{m-1}) \,, \tag{A.3}$$

$$\delta_\Omega^{(m)} V = h_{m+1} [\eta\Omega,\, (QV)^{m-1},\, V] + \mathcal{O}(Q^{m-2}) \,, \tag{A.4}$$

for $m \geq 1$, where g_{m+1} and h_{m+1} are coefficients to be determined.

Let us begin with the calculation of $(\delta_\Lambda S)_n$. It follows from (A.2), (A.3), $S_2 = \mathcal{O}(Q)$, and $\delta_\Lambda^{(0)} V = \mathcal{O}(Q)$ that the maximum number of Q's in $(\delta_\Lambda S)_n$ is $n+1$ for $n \geq 1$. The terms in $(\delta_\Lambda S)_n$ which contain $(n+1)$ Q's and those which contain n Q's are given by

$$\begin{aligned}
(\delta_\Lambda S)_n &= \sum_{m=0}^{n} \delta_\Lambda^{(m)} S_{n-m+2} \\
&= \Big[g_{n+1} - (n+2)\, f_{n+2} \Big] \{ Q\eta V,\, V,\, (QV)^{n-1},\, Q\Lambda \} + \\
&\quad + 2 \sum_{m=0}^{n-2} \left[f_{n+2,\,m} - \binom{n}{m} f_{n+2} \right] \{ \eta V,\, (QV)^m,\, Q\Lambda, [V,\, (QV)^{n-m-1}] \} + \\
&\quad + \sum_{m=1}^{n-1} \left[(m+2)\, g_{n-m+1}\, f_{m+2} - 2 \binom{n}{m} f_{n+2} - f_{n+2,\,n-m-1} - f_{n+2,\,m-1} \right] \times \\
&\quad \times \{ \eta V,\, (QV)^m, [V,\, (QV)^{n-m-1},\, Q\Lambda] \} + \\
&\quad + \langle Q\eta V,\, \mathcal{O}(Q^{n-1}) \rangle + \mathcal{O}(Q^{n-1}) \,,
\end{aligned} \tag{A.5}$$

where $\binom{n}{m}$ is the binomial coefficient $\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$. Gauge invariance requires

$$g_{n+1} - (n+2)\, f_{n+2} = 0 \,,$$

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$$f_{n+2,m} - \binom{n}{m} f_{n+2} = 0\,,$$

$$(m+2)\, g_{n-m+1}\, f_{m+2} - 2\binom{n}{m} f_{n+2} - f_{n+2,n-m-1} - f_{n+2,m-1} = 0\,. \tag{A.6}$$

These equations uniquely determine f_n, $f_{n,m}$, and g_n with $f_3 = 1/3!$:

$$f_n = \frac{1}{n!}\,, \qquad f_{n,m} = \frac{1}{n!}\binom{n-2}{m}\,, \qquad g_n = \frac{1}{n!}\,. \tag{A.7}$$

Let us next compute $(\delta_\Omega S)_n$. It follows from (A.2), (A.4), $S_2 = \mathcal{O}(Q)$, and $\delta_\Omega^{(0)} V = \mathcal{O}(Q^0)$ that the maximum number of Q's in $(\delta_\Omega S)_n$ is n for $n \geq 1$. The terms in $(\delta_\Omega S)_n$ which contain n Q's and the terms which contain $n-1$ Q's are given by

$$\begin{aligned}
(\delta_\Omega S)_n &= \sum_{m=0}^{n} \delta_\Omega^{(m)} S_{n-m+2} \\
&= \Big[h_{n+1} - n\,(n+2)\, f_{n+2}\Big]\{Q\eta V,\, \eta\Omega,\, (QV)^{n-1},\, V\} + \\
&\quad + \sum_{m=0}^{n-2}\left[(n-m)\, f_{n+2,m} - n\binom{n-1}{m} f_{n+2}\right] \times \\
&\quad \times \{\eta V, [\eta\Omega,\, (QV)^{n-m-1}]\,, (QV)^m,\, V\,\} + \\
&\quad + \sum_{m=0}^{n-3}\left[(m+1)\, f_{n+2,m+1} - n\binom{n-1}{m} f_{n+2}\right] \times \\
&\quad \times \{\eta V, [(QV)^{n-m-1}],\, \eta\Omega, (QV)^m,\, V\} + \\
&\quad + \sum_{m=1}^{n-1}\Bigg[(m+2)\, h_{n-m+1}\, f_{m+2} - 2\,n\binom{n-1}{m} f_{n+2} - \\
&\qquad\quad - (n-m)(f_{n+2,n-m-1} + f_{n+2,m-1})\Bigg] \times \\
&\quad \times \{\eta V, [\eta\Omega,\, (QV)^{n-m-1},\, V], (QV)^m\} + \langle Q\eta V,\, \mathcal{O}(Q^{n-2})\rangle + \mathcal{O}(Q^{n-2})\,.
\end{aligned} \tag{A.8}$$

Gauge invariance requires

$$\begin{aligned}
h_{n+1} - n\,(n+2)\, f_{n+2} &= 0\,, \\
(n-m)\, f_{n+2,m} - n\binom{n-1}{m} f_{n+2} &= 0\,, \\
(m+1)\, f_{n+2,m+1} - n\binom{n-1}{m} f_{n+2} &= 0\,, \\
(m{+}2)\, h_{n-m+1}\, f_{m+2} - 2\,n\binom{n-1}{m} f_{n+2} - (n-m)\,(\, f_{n+2,n-m-1} + f_{n+2,m-1}) &= 0\,.
\end{aligned} \tag{A.9}$$

These equations uniquely determine f_n, $f_{n,m}$, and h_n with $f_3 = 1/3!$:

$$f_n = \frac{1}{n!}\,, \qquad f_{n,m} = \frac{1}{n!}\binom{n-2}{m}\,, \qquad h_n = \frac{n-1}{n!}\,. \tag{A.10}$$

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The coefficients f_n and $f_{n,m}$ are in perfect agreement with those in (A.7). Since either one of the two sets of equations (A.6) and (A.9) is, by itself, an overdetermined system, it is highly nontrivial that there exists an action which respects gauge invariance under both $\delta_\Lambda V$ and $\delta_\Omega V$ up to this order.

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